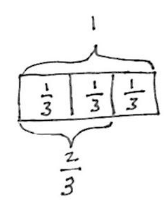
Grade 4 - Module 5: Fraction Equivalence, Ordering, and Operations

- Benchmark (standard or reference point by which something is measured)
- Common denominator (when two or more fractions have the same denominator)
- Denominator (bottom number in a fraction)
- Line plot (display of data on a number line, using an x or another mark to show frequency)
- Mixed number (number made up of a whole number and a fraction)
- Numerator (top number in a fraction)
- Compose (change a group of unit fractions with the same denominator to a single non-unit fraction or mixed number)
- Decompose (change a non-unit fraction or mixed number to the sum of its parts or unit fractions)
- Equivalent fractions (fractions that name the same size or amount)
- Fraction (e.g., 1/3, 2/3, 3/3, 4/3)
- Fraction greater than 1 (a fraction with a numerator that is greater than the denominator)
- Fractional unit (e.g., half, third, fourth)
- Unit fraction (fractions with numerator 1)

Topic A: Decomposition and Fraction Equivalence

Topic A builds on Grade 3 work with unit fractions. Students explore fraction equivalence through the decomposition of non-unit fractions into unit fractions, as well as the decomposition of unit fractions into smaller unit fractions. They represent these decompositions, and prove equivalence, using visual models.

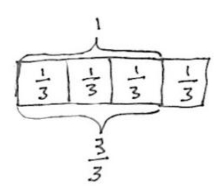
$$1 = \underbrace{\frac{1}{3} + \frac{1}{3}} + \frac{1}{3}$$
$$= \frac{2}{3} + \frac{1}{3}$$
$$= \frac{3}{3}$$



In Lessons 1 and 2, students decompose fractions as unit fractions, drawing tape diagrams to represent them as sums of fractions with the same denominator in different ways, e.g., – – – – – –.

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$
$$= 2 \times \frac{1}{3}$$

$$\frac{4}{3} = \frac{3}{3} + \frac{1}{3}$$
$$= 4 \times \frac{1}{3}$$



In Lesson 3, students see that representing a fraction as the repeated addition of a unit fraction is the same as multiplying that unit fraction by a whole number. This is already a familiar fact in other contexts. For example,

3 bananas = 1 banana + 1 banana + 1 banana = 3 × 1 banana,

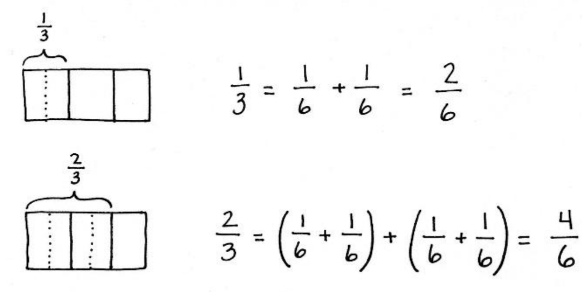
3 twos = 2 + 2 + 2 = 3 × 2

3 fourths = 1 fourth + 1 fourth + 1 fourth = 3 × 1 fourth,

– – – – –

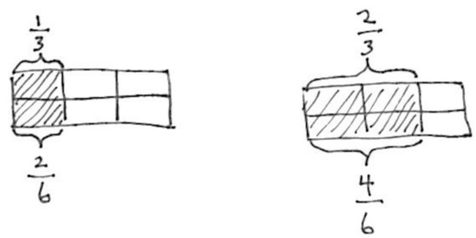
By introducing multiplication as a record of the decomposition of a fraction early in the module, students are accustomed to the notation by the time they work with more complex problems in Topic G.

Students continue with decomposition in Lesson 4, where they represent fractions, e.g., – , – , and – , as the sum of smaller unit fractions. They fold a paper strip to see that the number of fractional parts in a whole increases, while the size of the pieces decreases. Students investigate and verify this idea through a paper folding activity and record the results with tape diagrams, e.g.,



– – – – – – – –

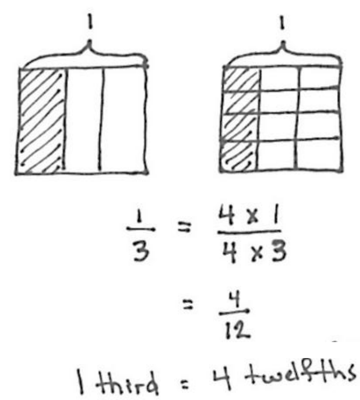
In Lesson 5, this idea is further investigated as students represent the decomposition of unit fractions in area models. In Lesson 6, students use the area model for a second day, this time to represent fractions with different numerators. They explain why two different fractions represent the same portion of a whole.



Topic B: Fraction Equivalence Using Multiplication and Division

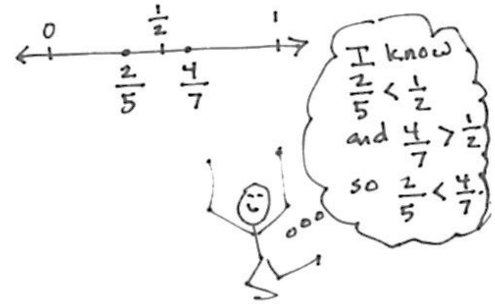
In Topic B, students start to generalize their work with fraction equivalence. In Lessons 7 and 8, students analyze their earlier work with tape diagrams and the area model in Lessons 3 through 5 to begin using multiplication to create an equivalent fraction comprised of smaller units, e.g., – –– — Conversely, students reason, in Lessons 9 and 10, that division can be used to create a fraction comprised of larger units (or a single unit) that is equivalent to a given fraction, e.g., — —— –. The numerical work of Lessons 7 through 10 is introduced and supported using area models and tape diagrams.

In Lesson 11, students use tape diagrams to transition their knowledge of fraction equivalence to the number line. They see that any unit fraction length can be partitioned into *n* equal lengths. For example, each third in the interval from 0 to 1 may be partitioned into 4 equal parts. Doing so multiplies both the total number of fractional units (the denominator) and the number of selected units (the numerator) by 4. On the other hand, students see that in some cases fractional units may be grouped together to form some number of larger fractional units. For example, when the interval from 0 to 1 is partitioned into twelfths, one may group 4 twelfths at a time to make thirds. In doing so, both the total number of fractional units and the number of selected units are divided by 4.

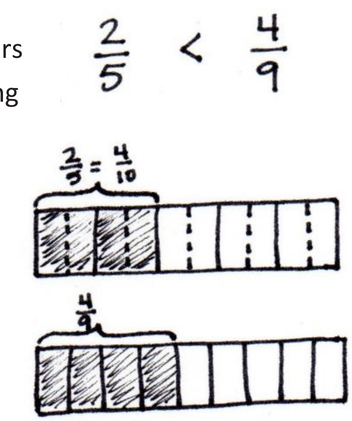


Topic C: Fraction Comparison

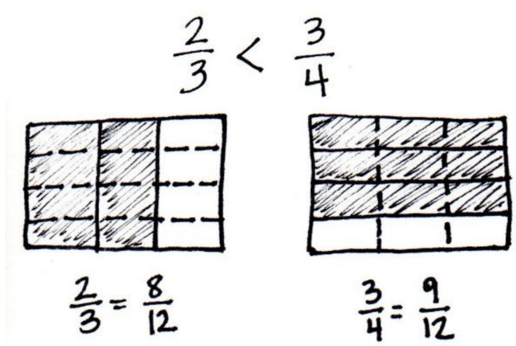
In Topic C, students use benchmarks and common units to compare fractions with different numerators and different denominators. The use of benchmarks is the focus of Lessons 12 and 13 and is modeled using a number line. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, – , or 1) and then use that information to compare the given fractions. For example, when comparing – and – , students reason that 4 sevenths is more than 1 half, while 2 fifths is less than 1 half. They then conclude that 4 sevenths is greater than 2 fifths.



In Lesson 14, students reason that they can also use like numerators based on what they know about the size of the fractional units. They begin at a simple level by reasoning, for example, that 3 fifths is less than 3 fourths because fifths are smaller than fourths. They then see, too, that it is easy to make like numerators at times to compare, e.g., – – because – — , and — – because — – Using their experience from fractions in Grade 3, they know the larger the denominator of a unit fraction, the smaller the size of the fractional unit. Like numerators are modeled using tape diagrams directly above each other, where one fractional unit is partitioned into smaller unit fractions. The lesson then moves to comparing fractions with related denominators, such as – and – , wherein one denominator is a factor of the other, using both tape diagrams and the number line.



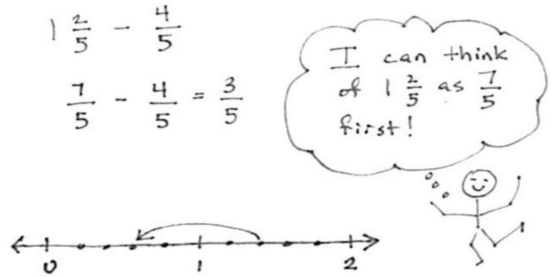
In Lesson 15, students compare fractions by using an area model to express two fractions, wherein one denominator is not a factor of the other, in terms of the same unit using multiplication, e.g., – – because – — — and – — — and — —. The area for – is partitioned vertically, and the area for – is partitioned horizontally.

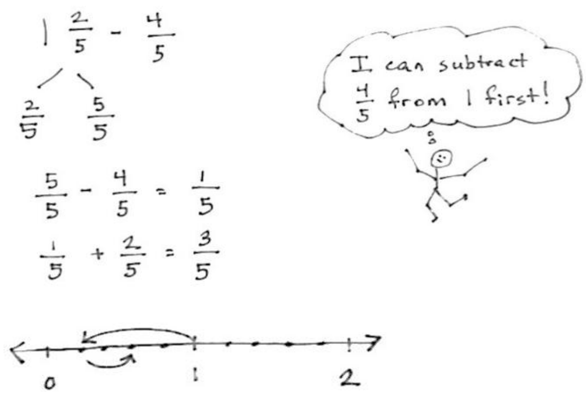


To find the equivalent fraction and to create the same size units, the areas are decomposed horizontally and vertically, respectively. Now the unit fractions are the same in each model or equation, and students can easily compare. The topic culminates with students comparing pairs of fractions and, in so doing, deciding which strategy is either necessary or efficient: reasoning using benchmarks and what they know about units, drawing a model such as number line, tape diagram, or area model, or the general method of finding like denominators through multiplication.

Topic D: Fraction Addition and Subtraction

Topic D bridges students' understanding of whole number addition and subtraction to fractions. Everything that they know to be true of addition and subtraction with whole numbers now applies to fractions. Addition is finding a total by combining like units. Subtraction is finding an unknown part. Implicit in the equations 3 + 2 = 5 and 2 = 5 – 3 is the assumption that the numbers are referring to the *same* units.

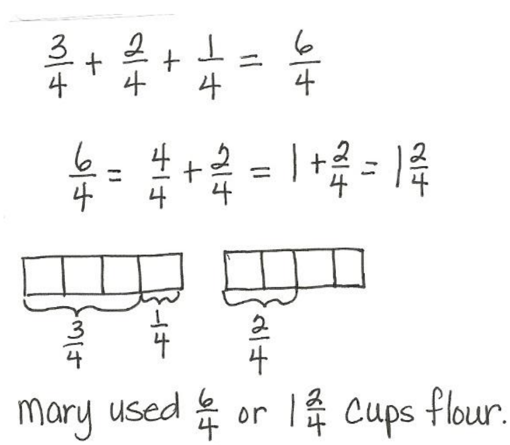


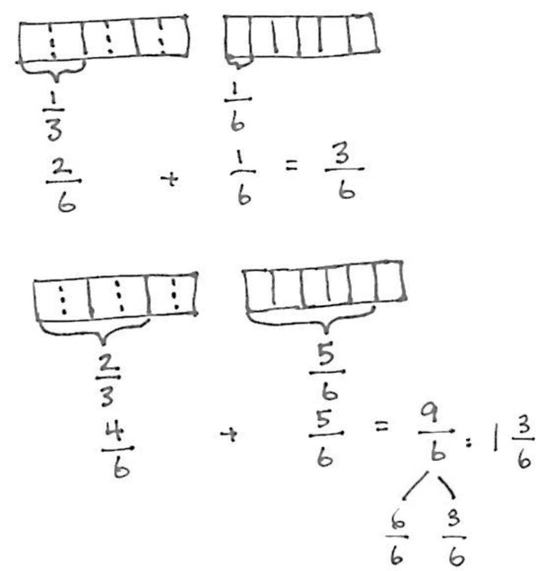


In Lessons 16 and 17, students generalize familiar facts about whole number addition and subtraction to work with fractions. Just as 3 apples – 2 apples = 1 apple, students note that 3 fourths – 2 fourths = 1 fourth. Just as 6 days + 3 days = 9 days = 1 week 2 days, students note that – – – – – –. In Lesson 17, students decompose a whole into a fraction having the same denominator as the subtrahend. For example, 1 – 4 fifths becomes 5 fifths – 4 fifths = 1 fifth, connecting with Topic B skills. They then see that when solving – – –, they have a choice of subtracting – from – or from 1 (as pictured to the right). Students model with tape diagrams and number lines to understand and then verify their numerical work.

In Lesson 18, students add more than two fractions and see sums of more than one whole, such as – – – – — As students move into problem solving in Lesson 19, they create tape diagrams or number lines to represent and solve fraction addition and subtraction word problems (see example below). These problems bridge students into work with mixed numbers to follow the Mid-Module Assessment.

Mary mixed – cup of wheat flour, – cup of rice flour, and – cup of oat flour for her bread dough. How many cups of flour did she put in her bread in all?



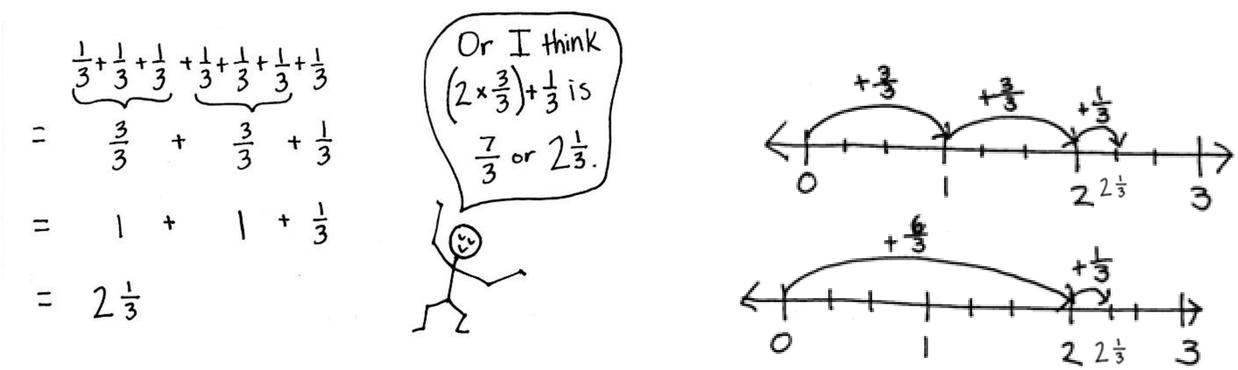


In Lessons 20 and 21, students add fractions with related units, where one denominator is a multiple (or factor) of the other. In order to add such fractions, a decomposition is necessary. Decomposing one unit into another is familiar territory: Students have had ample practice composing and decomposing in Topics A and B when working with place value units, when converting units of measurement, and when using the distributive property. For example, they have converted between equivalent measurement units (e.g., 100 cm = 1 m), and they've used such conversions to do arithmetic (e.g., 1 meter – 54 centimeters). With fractions, the concept is the same. To find the sum of – and – , one simply renames (converts, decomposes) – as – and adds: – – –. All numerical work is accompanied by visual models that allow students to use and apply their known skills and understandings. Number sentences involve the related units of 2, 4 and 8, 2 and 10, 3 and 6, and 5 and 10. The addition of fractions with related units is also foundational to decimal work when adding tenths and hundredths in Module 6. Please note that addition of fractions with related denominators will not be assessed.

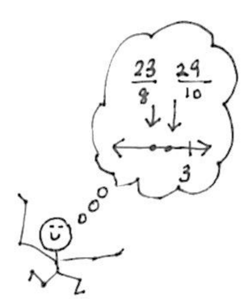
Topic E: Extending Fraction Equivalence to Fractions Greater than 1

In Topic E, students study equivalence involving both ones and fractional units. In Lesson 22, they use decomposition and visual models to add and subtract fractions less than 1 to and from whole numbers, e.g., 4 + – = – and 4 – – = (3 + 1) – – , subtracting the fraction from 1 using a number bond and a number line.

Lesson 23 has students using addition and multiplication to build fractions greater than 1 and then representing them on the number line. Fractions can be expressed both in mixed units of a whole number and a fraction or simply as a fraction, as pictured below, e.g., – – – – – – – –.



In Lessons 24 and 25, students use decompositions to reason about the various equivalent forms in which a fraction greater than or equal to 1 may be presented: both as fractions and as mixed numbers. In Lesson 24, they decompose, for example, 11 fourths into 8 fourths and 3 fourths, — – – , or they can think of it as — – – – = 2 – – –. In Lesson 25, students are then able to decompose the two wholes into 8 fourths so their original number can now be looked at as – – — In this way, they see that – —. This fact is further reinforced when they plot — on the number line and see that it is at the same point as –. Unfortunately, the term *improper fraction* carries with it some baggage. As many have observed, there is nothing "improper" about an improper fraction. Nevertheless, as a mathematical term, it is useful for describing a particular form in which a fraction may be presented (i.e., a fraction is improper if the numerator is greater than or equal to the denominator). Students do need practice in converting between the various forms a fraction may take, but take care not to foster the misconception that every improper fraction *must* be converted to a mixed number.

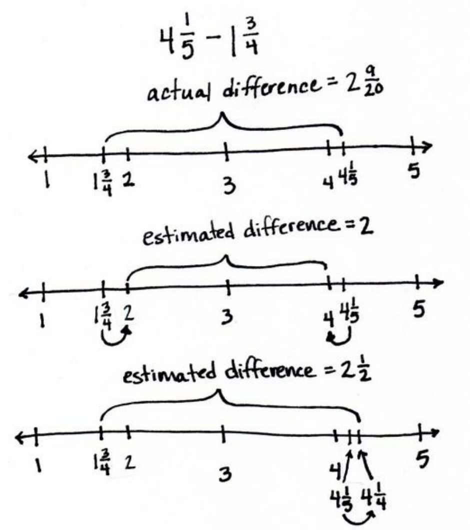


Students compare fractions greater than 1 in Lessons 26 and 27. They begin by using their understanding of benchmarks to reason about which of two fractions is greater. This activity builds on students' rounding skills, having them identify the whole numbers and the halfway points between them on the number line. The relationship between the numerator and denominator of a fraction is a key concept here as students consider relationships to whole numbers, e.g., a student might reason that — is less than— because — is 1 eighth less than 3, but — is 1 tenth less than . They know each fraction is 1 fractional unit away from 3 and since – > — then — < —. Students progress to finding and using like denominators to compare and order mixed numbers. Once again, students must use reasoning skills as they determine that when they have two fractions with the same numerator, the larger fraction will have a larger unit (or smaller denominator). Conversely, when they have two fractions with the same denominator, the larger one will have the larger number of units (or larger numerator).

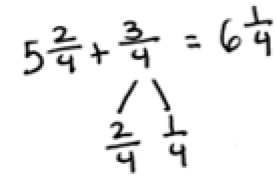
Lesson 28 wraps up the topic with word problems requiring the interpretation of data presented in line plots. Students create line plots to display a given dataset that includes fraction and mixed number values. To do this, they apply their skill in comparing mixed numbers, both through reasoning and through the use of common numerators or denominators. For example, a dataset might contain both – and — giving students the opportunity to determine that they must be plotted at the same point. They also use addition and subtraction to solve the problems.

Topic F: Addition and Subtraction of Fractions by Decomposition

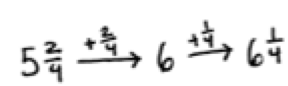
Topic F provides students with the opportunity to use their understandings of fraction addition and subtraction as they explore mixed number addition and subtraction by decomposition.



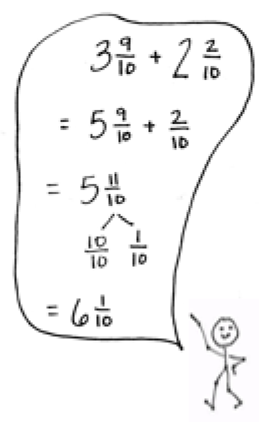
Lesson 29 focuses on the process of using benchmark numbers to estimate sums and differences of mixed numbers. Students once again call on their understanding of benchmark fractions as they determine, prior to performing the actual operation, what a reasonable outcome will be. One student might use benchmark whole numbers and reason, for example, that the difference between – and – is close to 2 because – is closer to 4 than 5, – is closer to 2 than 1, and the difference between 4 and 2 is 2. Another student might use familiar benchmark fractions and reason that the answer will be closer to – since – is about – more than 4 and – is about – less than 2, making the difference about a half more than 2 or –.



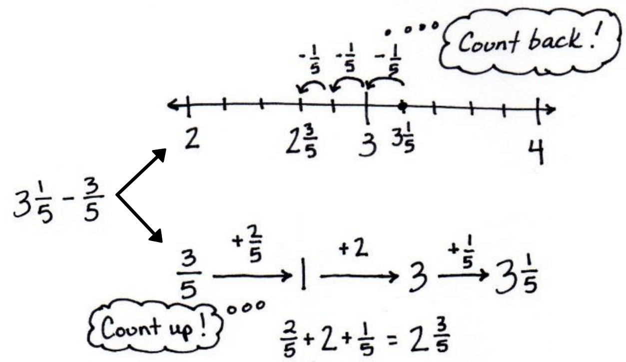
In Lesson 30, students begin adding a mixed number to a fraction using unit form. They add like units, applying their Grade 1 and 2 understanding of completing a unit to add when the sum of the fractional units exceeds 1. Students ask, “How many more do we need to make one?” rather than “How many more do we need to make ten?” as was the case in Grade 1. A number bond decomposes the fraction to make one and can be modeled on the number line or using the arrow way, as shown to the right. Alternatively, a number bond can be used after adding like units, when the sum results in a mixed number with a fraction greater than 1, to decompose the fraction greater than 1 into ones and fractional units.



Directly applying what was learned in Lesson 30, Lesson 31 starts with adding like units, ones with ones and fourths with fourths, to add two mixed numbers. Students can, again, choose to make one before finding the sum or to decompose the sum to result in a proper mixed number.



Lessons 32 and 33 follow the same sequence for subtraction. In Lesson 32, students simply subtract a fraction from a mixed number, using three main strategies both when there are and when there are not enough fractional units. They count back or up, subtract from 1, or take one out to subtract from 1. In Lesson 33, students apply these strategies after subtracting the ones first. They model subtraction of mixed numbers using a number line or the arrow way.



$$3\frac{1}{5} - \underbrace{\frac{3}{5}}_{\frac{1}{5}\ \ \frac{2}{5}} = 3 - \frac{2}{5} = 2\frac{3}{5}$$

Just like subtracting from one!

$$\underbrace{3\frac{1}{5}}_{2\frac{1}{5}\ \ 1} - \frac{3}{5} = 2\frac{1}{5} + \frac{2}{5} = 2\frac{3}{5}$$

Take one out to subtract from one!

In Lesson 34, students learn another strategy for subtraction by decomposing the total into a mixed number and an improper fraction to either subtract a fraction or a mixed number.

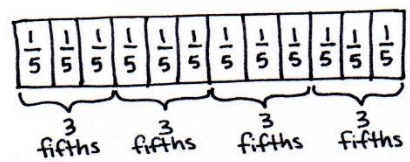
$$\underbrace{8\frac{1}{10}}_{7\ \ \frac{11}{10}} - \frac{8}{10} = 7\frac{11}{10} - \frac{8}{10} = 7\frac{3}{10}$$

$$11\frac{1}{5} - 2\frac{3}{5} = \underbrace{9\frac{1}{5}}_{8\ \ \frac{6}{5}} - \frac{3}{5} = 8\frac{3}{5}$$

Topic G: Repeated Addition of Fractions as Multiplication

Topic G extends the concept of representing repeated addition as multiplication, applying this familiar concept to work with fractions.

Multiplying a whole number times a fraction was introduced in Topic A as students learned to decompose fractions, e.g., – = –   –   – = 3 × –. In Lessons 35 and 36, students use the associative property, as exemplified below, to multiply a whole number times a mixed number.



3 bananas + 3 bananas + 3 bananas + 3 bananas

= 4   3 bananas

= 4   (3   1 banana) = (4   3)   1 banana = 12 bananas

3 fifths + 3 fifths + 3 fifths + 3 fifths

= 4   3 fifths

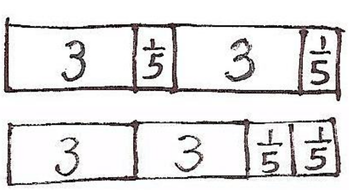
= 4   (3 fifths) = (4   3) fifths= 12 fifths

$4 \times (3 \text{ fifths}) = (4 \times 3) \text{ fifths}$
$= 12 \text{ fifths}$

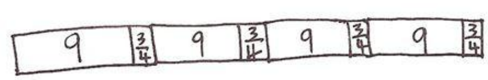
$4 \times 3 \text{ fifths} = 12 \text{ fifths}$
$4 \times \frac{3}{5} = \frac{12}{5}$

Students may never have considered before that 3 bananas = 3   1 banana, but it is an understanding that connects place value, whole number work, measurement conversions, and fractions, e.g., 3 hundreds = 3   1 hundred, or 3 feet = 3   (1 foot); 1 foot = 12 inches, therefore, 3 feet = 3   (12 inches) = (3   12) inches = 36 inches.

Students explore the use of the distributive property in Lessons 37 and 38 to multiply a whole number by a mixed number. They see the multiplication of each part of a mixed number by the whole number and use the appropriate strategies to do so. As students progress through each lesson, they are encouraged to record only as much as they need to keep track of the math. As shown below, there are multiple steps when using the distributive property, and students can get lost in those steps. Efficiency in solving is encouraged.



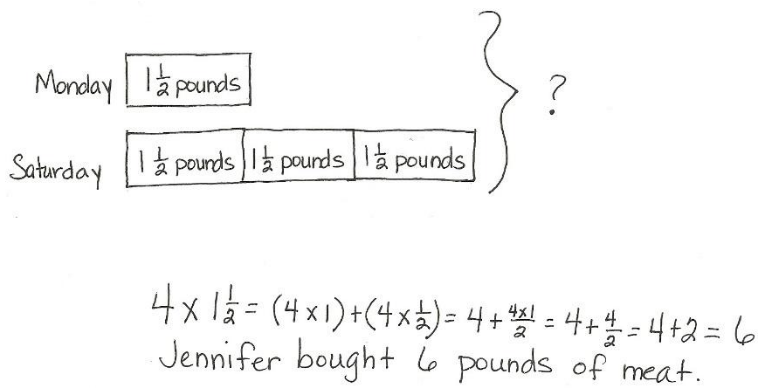
$2 \times 3\frac{1}{5} = (2 \times 3) + (2 \times \frac{1}{5})$
$= 6 + \frac{2}{5} = 6\frac{2}{5}$



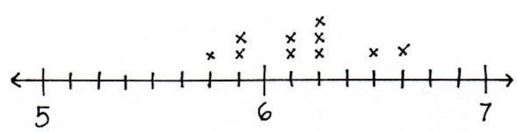
$4 \times 9\frac{3}{4} = 36 + \frac{12}{4}$
$= 36 + 3$
39

$5 \times 3\frac{3}{4} = 5 \times (3 + \frac{3}{4}) = (5 \times 3) + (5 \times \frac{3}{4}) = 15 + \frac{5 \times 3}{4} = 15 + \frac{15}{4} = 15 + 3\frac{3}{4} = 18\frac{3}{4}$

In Lesson 39, students build their problem-solving skills by solving multiplicative comparison word problems involving mixed numbers, e.g., “Jennifer bought 3 times as much meat on Saturday as she did on Monday. If she bought – pounds on Monday, how much did she buy on both days?” They create and use tape diagrams to represent these problems before using various strategies to solve them numerically.



In Lesson 40, students solve word problems involving multiplication of a fraction by a whole number and also work with data presented in line plots.



Topic H: Explore a Fraction Pattern

The final topic is an exploration lesson in which students find the sum of all like denominators from – –. For example, they might find the sum of all fifths from – to – Students discover they can make pairs with a sum of 1 to add more efficiently, e.g., – – – – – – As they make this discovery, they share and compare their strategies with partners. Through discussion of their strategies, they determine which are most efficient.

Next, students extend the use of their strategies to find sums of eighths, tenths, and twelfths, observing patterns when finding the sum of odd and even denominators (**4.OA.5**). Advanced students can be challenged to find the sum of all hundredths from 0 hundredths to 100 hundredths.